sets

set: unordered collection of objects.

$\downarrow$ can be anything (including other sets)

$\{\underbrace{0}_{1}, \underbrace{2}_{2}, \underbrace{\{1,3\}}_{3}\} = \{0,2,0,\{1,3\}\} = \{2,\{1,3\},0\}$

ordering and repeated elements do not change the set.

how to describe a set:

a) "set of red fruits"

b) {apple, strawberry, cherry ... }

c) $\{\boxed{x} \in \text{fruits} \mid x \text{ is red}\}$ ☆ set builder notation ☆

$\mid$ $\downarrow$ such that

set of fruits x such that each x is red.

cardinality: the number of unique elements in the set

set A, cardinality of A is $|A|$

$|\{1,2,3\}| = 3$ $\quad$ $|\{\underbrace{\{1,2\}}, \underbrace{3}\}| = 2$

• note, same symbol as absolute val, but A's type will tell you which operation we mean

$|\{1,1,1\}| = 1$

subsets

let A and B be sets.

$A \subseteq B$ iff all elements of A are also in B

$\{1,2\} \subseteq \{3,2,4,1\}$

$A \subset B$ means A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$.

* note: $\{\}$ empty set
$\emptyset$ or, null set.

$\emptyset \subseteq A$ for all sets A

$$\underbrace{\text{if } \underbrace{\text{element } x \in \emptyset}_{\text{always false}} \text{ then } x \in A}_{\text{always true}} = \emptyset \subseteq A$$

(remember truth table for implication)

vacuously true for $\emptyset \subseteq A$

but, $\emptyset \notin A$ for all sets A

$\in$ means "is an element of"

ex) $A = \{1,2,3\}$
$\emptyset \notin A$, but $\emptyset \in \{\emptyset, 1, 2\}$

$B = \{\{\emptyset\}\}$ $\quad \emptyset \notin B$
$\{\emptyset\} \in B$

operations

$A \cap B$ intersection

$A = \{\text{green}, \text{blue}\}$
$B = \{\text{blue}, \text{orange}\}$

$A \cap B = \{s \mid s \in A \text{ and } s \in B\}$

$A \cap B = \{\text{blue}\}$

☆ if $A \cap B = \emptyset$ (if they have nothing in common) the sets are called disjoint

$A \cup B$ union

$A \cup B = \{s \mid s \in A \text{ or } s \in B\}$

$A \cup B = \{\text{green}, \text{blue}, \text{orange})$

$A - B$ difference

$A - B = \{s \mid s \in A \text{ but } s \notin B\}$

$A - B = \{\text{green}\}$

$A \times B$ product, cross product, cartesian product, "cross"

$A \times B = \{(a, b) \mid a \in A, b \in B\}$

$A \times B = \{(\text{green}, \text{blue}), (\text{green}, \text{orange}), (\text{blue}, \text{blue}), (\text{blue}, \text{orange})\}$

this order matters, they are ordered pairs

Proof: show $A \subseteq B$. (concrete)

ex) Prove $A \subseteq B$ where $A = \{\lambda(2,3) + (1-\lambda)(7,4) \mid \lambda \in [0,1]\}$

$B = \{(x,y) \mid x,y \in \mathbb{R}, x \geq 0, y \geq 0\}$.

ordered pairs.

First, let's simplify the def'n of A.

$A = \{\lambda(2,3) + (1-\lambda)(7,4) \mid \lambda \in [0,1]\}$ $\in\mathbb{R}$

$\{(2\lambda, 3\lambda) + (7-7\lambda, 4-4\lambda) \mid \lambda \in \mathbb{R}, \in [0,1]\}$

$\{(2\lambda+7-7\lambda, 3\lambda+4-4\lambda) \mid \lambda \in \mathbb{R}, \in [0,1]\}$

$A = \{(7-5\lambda, 4-\lambda) \mid \lambda \in \mathbb{R}, \in [0,1]\}$

Then pick a representative element of A. Let's say $(c,d) \in A$.

Then, by def'n of A, $c = 7-5\lambda$ where $\lambda \in \mathbb{R}, \in [0,1]$.
$d = 4-\lambda$

This means $\lambda \leq 1$

$c = 7-5\lambda$
$\lambda \leq 1$
$-5\lambda \geq -5$
$7-5\lambda \geq 7-5 = 2$
$c \geq 2 \geq 0$
$c \geq 0$

$d = 4-\lambda$
$\lambda \leq 1$
$-\lambda \geq -1$
$4-\lambda \geq 4-1 = 3$
$4-\lambda \geq 3$
$d \geq 3 \geq 0$
$d \geq 0$

now, we have element $(c,d) \in \mathbb{R}^2$, $c \geq 0$ and $d \geq 0$.

This element, by def'n of B, is also an element of B.

So $A \subseteq B$.